Lesson 6. Sets, Summations, For Statements

1 Sets

• A set is a collections of elements/objects, e.g.

$$S = \{1, 2, 3, 4, 5\}$$
 Fruits = {Apple, Orange, Pear} (1)

• "in" symbol:

$$i \in N \iff$$
 "element i is in the set N "

• For example:

2 Summations

• Summation symbol over sets:

$$\sum_{i \in N} \Leftrightarrow \text{"sum over all elements of } N$$
"

• For example:

• Common shorthand: if $N = \{1, 2, ..., n\}$, then

$$\sum_{i \in N} \text{ is the same as } \sum_{i \in \{1,2,\dots,n\}} \text{ as well as } \sum_{i=1}^{n}$$

Example 1. Let the sets *S* and Fruits be defined as above in (1). Write each of the following as compactly as possible using summation notation:

a.
$$x_{\text{Apple}} + x_{\text{Orange}} + x_{\text{Pear}}$$

b.
$$1y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5$$

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• "for" statements over sets:

for $i \in N \iff$ "repeat for each element of N"

• For example:

$$c_j x_1 + d_j x_2 \le b_j$$
 for $j \in \text{Fruits} \iff$

• Common shorthand: if $N = \{1, 2, ..., n\}$, then

"for
$$i \in N$$
" is the same as "for $i \in \{1, 2, ..., n\}$ " as well as "for $i = 1, 2, ..., n$ "

• Sometimes we say "for all $i \in N$ " instead of "for $i \in N$ "

4 Multiple indices

- Sometimes it may be useful to use decision variables with multiple indices
- Example:
 - \circ Set of hat types: $H = \{A, B, C\}$
 - Set of factories: $F = \{1, 2\}$
 - Each hat type can be be produced at each factory
 - o Define decision variables:

$$x_{i,j}$$
 = number of type i hats produced at factory j for $i \in H$ and $j \in F$ (2)

• What decision variables have we just defined? How many are there?



Example 2. Using the decision variables defined in (2), write expressions for

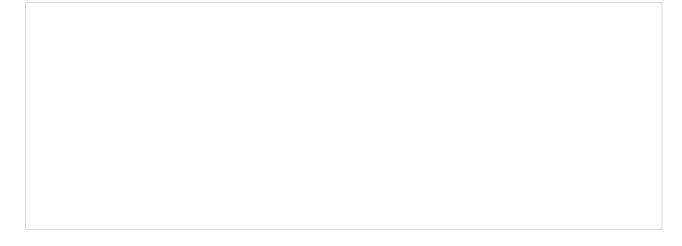
- a. Total number of type *C* hats produced
- b. Total number of hats produced at facility 2

Use summation notation if possible.

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 $c_{i,j}$ = cost of producing one type i hat at factory j for $i \in H$ and $j \in F$

• If we produce $x_{i,j}$ hats of type i at factory j (for $i \in H$ and $j \in F$), then the total cost is



Example 3. Let $M = \{1, 2, 3\}$ and $N = \{1, 2, 3, 4\}$. Write the following as compactly as possible using summation notation and "for" statements.

Let y_1 = amount of product 1 produced

 y_2 = amount of product 2 produced

 y_3 = amount of product 3 produced

 y_4 = amount of product 4 produced

$$a_{1,1}y_1 + a_{1,2}y_2 + a_{1,3}y_3 + a_{1,4}y_4 = b_1$$

$$a_{2,1}y_1 + a_{2,2}y_2 + a_{2,3}y_3 + a_{2,4}y_4 = b_2$$

 $a_{3,1}y_1 + a_{3,2}y_2 + a_{3,3}y_3 + a_{3,4}y_4 = b_3$